

Limit on the Color-Triplet Higgs Mass in the Minimum Supersymmetric SU(5) Model

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Abstract

In the minimum supersymmetric SU(5) GUT, we derive the upper limit on the mass of the color-triplet Higgs multiplets as $M_{H_c} \leq 2.4 \times 10^{16}$ GeV (90 % C.L.) taking all possible corrections into account in a renormalization group analysis. If the above upper limit is compared with a limit on M_{H_c} from the negative search for the proton decay; $M_{H_c} \geq 2.0 \times 10^{16}$ GeV (in which effects of the larger top-quark mass are included), the minimum supersymmetric SU(5) GUT is severely constrained.

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The supersymmetric grand unified theory (SUSY-GUT) attracts us as a candidate of physics beyond the standard model (SM). The phenomenological successes of the SUSY-GUT are not only the gauge coupling constant unification [1], but also the m_b/m_τ ratio [2]. On the other hand, the proton decay, which is a direct evidence of GUT, has not yet been observed. In the minimum SUSY-SU(5) GUT [3], the exchanges of color-triplet Higgs multiplets give rise to the most dominant contribution to the nucleon decay [4] whose amplitudes depend on an unknown GUT-scale parameter, that is the mass of the color-triplet Higgs multiplets M_{H_c} [5, 6]. Therefore, the negative search for the nucleon decay constrains the model strongly, and hence it is very important to determine the mass M_{H_c} without any theoretical prejudices in order to check the consistency of this model.

It has been shown in Ref.[7] that the color-triplet Higgs mass M_{H_c} can be determined in the minimum SUSY-SU(5) model by using only the low-energy parameters, *i.e.*, the gauge coupling constants and the superparticle mass spectrum at the electroweak scale. However, in the previous analysis [7] the effects of the two-loop corrections below the sfermion mass scale ($\simeq 1$ TeV) and the one-loop finite threshold corrections of the SUSY-particles have not been taken into account. In this letter, we improve the analysis by including these effects as well as by taking the effect of the top-quark Yukawa coupling [8] into account and by using the most recent experimental data on the gauge coupling constants [9]. As a result, we obtain the upper limit on M_{H_c} as $M_{H_c} \leq 2.4 \times 10^{16}$ GeV (90% C.L.). We have also checked that the correction from the $\Sigma(\mathbf{24})$ loop does not change this result much even if there exists a large mass splitting among the superheavy particles.

Furthermore, the lower limit on M_{H_c} from the negative search for the proton decay [5] is also affected by the low-energy experimental data. Recently, CDF collaboration reported the evidence of top-quark production with its mass $174 \pm 10^{+13}_{-12}$ GeV [10]. Thus, we need to reanalyze the constraints on M_{H_c} since the top-quark mass has been assumed at 90 GeV in the previous analysis [5]. When this lower limit on M_{H_c} from the negative search for the proton decay is compared with the above upper limit on M_{H_c} derived from the renormalization group (RG) analysis, we find that the minimum SUSY-SU(5) GUT is severely constrained and we conclude that the minimum SUSY-SU(5) model is consistent only in a very narrow parameter region.

Let us start with studying the minimum SUSY-SU(5) model. This model contains the following chiral supermultiplets;

$$\text{matter : } \psi_i(\mathbf{10}), \quad \phi_i(\mathbf{\bar{5}}), \quad \text{Higgs : } H(\mathbf{5}), \quad \bar{H}(\mathbf{\bar{5}}), \quad \Sigma(\mathbf{24}), \quad (1)$$

where $i(=1,2,3)$ represents the family index. The superpotential in this model is

$$W = \frac{f}{3} \text{Tr} \Sigma^3 + \frac{1}{2} f V \text{Tr} \Sigma^2 + \lambda \overline{H}_A (\Sigma_B^A + 3V \delta_B^A) H^B + \frac{h^{ij}}{4} \epsilon_{ABCDE} \psi_i^{AB} \psi_j^{CD} H^E + \sqrt{2} f^{ij} \psi_i^{AB} \phi_{jA} \overline{H}_B, \quad (2)$$

where indices A, B, C, \dots are the SU(5) indices which run from 1 to 5 and ϵ_{ABCDE} the fifth-antisymmetric tensor.

The **24**-dimension Higgs $\Sigma(\mathbf{24})$ has the following vacuum-expectation value that causes the breaking $\text{SU}(5) \rightarrow \text{SU}(3)_C \times \text{SU}(2)_L \times \text{U}(1)_Y$,

$$\langle \Sigma \rangle = V \begin{pmatrix} 2 & & & & \\ & 2 & & & \\ & & 2 & & \\ & & & -3 & \\ & & & & -3 \end{pmatrix}. \quad (3)$$

This vacuum-expectation value gives the following masses to X and Y gauge bosons corresponding to the broken SU(5) generators;

$$M_V = M_X = M_Y = 5\sqrt{2}g_5 V, \quad (4)$$

where g_5 is the unified SU(5) gauge coupling constant. The invariant mass parameter of $H(\mathbf{5})$ and $\overline{H}(\overline{\mathbf{5}})$ is fine-tuned to realize massless SU(2)_L-doublet Higgs multiplets H_f and \overline{H}_f while their color-triplet partners H_c and \overline{H}_c are kept superheavy as

$$M_{H_c} = M_{\overline{H}_c} = 5\lambda V. \quad (5)$$

After the SU(5) symmetry breaking, $\Sigma(\mathbf{24})$ is decomposed into various mass eigenstates. The masses $M_{\mathbf{8}}$ and $M_{\mathbf{3}}$ for the components $(\mathbf{8}, \mathbf{1})$ and $(\mathbf{1}, \mathbf{3})$ under $\text{SU}(3)_C \times \text{SU}(2)_L$ are given by

$$M_{\Sigma} \equiv M_{\mathbf{8}} = M_{\mathbf{3}} = \frac{5}{2} f V, \quad (6)$$

while the components $(\mathbf{3}, \mathbf{2})$ and $(\overline{\mathbf{3}}, \mathbf{2})$ form superheavy vector multiplets of mass M_V together with the gauge multiplets X and Y , and the mass of singlet component $(\mathbf{1}, \mathbf{1})$ is $\frac{1}{2} f V = \frac{1}{5} M_{\Sigma}$ [5].

Before performing numerical analyses, let us briefly review the procedure we use. The masses M_V , M_{Σ} , and M_{H_c} for the superheavy particles are constrained from the low-energy parameters, *i.e.* the gauge coupling constants and the superparticle mass spectrum, as

shown in Ref.[7]. Requiring the unification condition, we can relate three gauge coupling constants at $\mu = m_Z$ (with μ being a renormalization point) to the SU(5) gauge coupling constant $\alpha_5 = g_5^2/4\pi$ at $\mu = \Lambda \gg M_V, M_\Sigma$, and M_{H_c} . At the one loop level, the relations are given by²

$$\alpha_3^{-1}(m_Z) = \alpha_5^{-1}(\Lambda) + \frac{1}{2\pi} \left\{ \left(-2 - \frac{2}{3}N_g \right) \ln \frac{m_{\text{SUSY}}}{m_Z} + (-9 + 2N_g) \ln \frac{\Lambda}{m_Z} - 4 \ln \frac{\Lambda}{M_V} + 3 \ln \frac{\Lambda}{M_\Sigma} + \ln \frac{\Lambda}{M_{H_c}} \right\}, \quad (7)$$

$$\alpha_2^{-1}(m_Z) = \alpha_5^{-1}(\Lambda) + \frac{1}{2\pi} \left\{ \left(-\frac{4}{3} - \frac{2}{3}N_g - \frac{5}{6} \right) \ln \frac{m_{\text{SUSY}}}{m_Z} + (-6 + 2N_g + 1) \ln \frac{\Lambda}{m_Z} - 6 \ln \frac{\Lambda}{M_V} + 2 \ln \frac{\Lambda}{M_\Sigma} \right\}, \quad (8)$$

$$\alpha_1^{-1}(m_Z) = \alpha_5^{-1}(\Lambda) + \frac{1}{2\pi} \left\{ \left(-\frac{2}{3}N_g - \frac{1}{2} \right) \ln \frac{m_{\text{SUSY}}}{m_Z} + \left(2N_g + \frac{3}{5} \right) \ln \frac{\Lambda}{m_Z} - 10 \ln \frac{\Lambda}{M_V} + \frac{2}{5} \ln \frac{\Lambda}{M_{H_c}} \right\}, \quad (9)$$

where N_g represents the number of the families. In Eqs.(7) – (9) we have assumed that all superparticles in the SUSY-standard model have a common SUSY-breaking mass, m_{SUSY} , for simplicity. By taking the suitable linear combinations of Eqs.(7) – (9), we obtain simple relations,

$$(3\alpha_2^{-1} - 2\alpha_3^{-1} - \alpha_1^{-1})(m_Z) = \frac{1}{2\pi} \left\{ \frac{12}{5} \ln \frac{M_{H_c}}{m_Z} - 2 \ln \frac{m_{\text{SUSY}}}{m_Z} \right\}, \quad (10)$$

$$(5\alpha_1^{-1} - 3\alpha_2^{-1} - 2\alpha_3^{-1})(m_Z) = \frac{1}{2\pi} \left\{ 12 \ln \frac{M_V^2 M_\Sigma}{m_Z^3} + 8 \ln \frac{m_{\text{SUSY}}}{m_Z} \right\}. \quad (11)$$

These equations imply that we can give the constraint to the superheavy masses from the precision data of α_1, α_2 and α_3 at the electroweak scale.

In a qualitative analysis, the limit on M_{H_c} depends on the mass spectrum of the superparticles. The effect of the mass splittings among superpartners can be taken into account by replacing $\ln(m_{\text{SUSY}}/m_Z)$ in Eq.(10) as

$$-2 \ln \frac{m_{\text{SUSY}}}{m_Z} \rightarrow 4 \ln \frac{m_{\tilde{g}}}{m_{\tilde{w}}} + \frac{N_g}{5} \ln \frac{m_{\tilde{u}}^3 m_d^2 m_{\tilde{e}}}{m_{\tilde{Q}}^4 m_L^2} - \frac{8}{5} \ln \frac{m_{\tilde{h}}}{m_Z} - \frac{2}{5} \ln \frac{m_H}{m_Z}, \quad (12)$$

and that in Eq.(11) as

$$8 \ln \frac{m_{\text{SUSY}}}{m_Z} \rightarrow 4 \ln \frac{m_{\tilde{g}}}{m_Z} + 4 \ln \frac{m_{\tilde{w}}}{m_Z} + N_g \ln \frac{m_{\tilde{Q}}^2}{m_{\tilde{e}} m_{\tilde{u}}}. \quad (13)$$

²The U(1)_Y gauge coupling constant is normalized such as $\alpha_Y = \frac{3}{5}\alpha_1$.

Here, two doublet Higgs bosons are assumed to have masses at m_H and m_Z , respectively. The symbols $m_{\tilde{q}}$ ($\tilde{q} = \tilde{Q}, \tilde{u}, \tilde{d}$), $m_{\tilde{l}}$ ($\tilde{l} = \tilde{L}, \tilde{e}$), $m_{\tilde{w}}$, $m_{\tilde{g}}$, and $m_{\tilde{h}}$ represent the masses of squarks, sleptons, wino, gluino and doublet Higgsinos. From Eq.(10) and Eq.(12), one can see that the M_{H_c} increases as $m_{\tilde{h}}$ and m_H become larger. In order to derive a conservative upper limit on M_{H_c} , we should use the maximal values of $m_{\tilde{h}}$ and m_H allowed from the naturalness point of view. In our analysis, we take $m_{\tilde{h}} = m_H = 1\text{TeV}$. The mass ratio $m_{\tilde{g}}/m_{\tilde{w}}$ is given by α_3/α_2 because of the unification of the gaugino masses at the GUT scale, and hence the first term in the right-handed side of Eq.(12) gives a constant independent of the gaugino masses. Assuming that all the sfermion masses are universal at the GUT scale, the second term takes the minimal value if the gaugino mass is much smaller than the sfermion masses (see Ref.[5, 11]). Thus, the case where the sfermion masses are heavier than the gaugino masses gives the conservative upper limit on M_{H_c} at the one-loop level. Though we have assumed that the sfermion masses for $\phi(\mathbf{\bar{5}})$ and $\psi(\mathbf{10})$ are the same at the GUT scale, they may be different each other. In the situation that the sfermion masses for $\psi(\mathbf{10})$ are comparable to gaugino masses at the electroweak scale while the ones for $\phi(\mathbf{\bar{5}})$ are heavier than them, the upper limit on M_{H_c} is raised by factor 1.6. However, this choice of the mass spectrum makes the proton lifetime much shorter, and hence we do not consider this situation. From these arguments, we take the wino mass smaller than 200 GeV,³ and all sfermion masses at 1 TeV. It should be noted here that the negative search for the proton decay prefers this situation.

As the input parameters, we use the most recent values of the $\overline{\text{MS}}$ (a modified minimal subtraction) gauge coupling constants at the Z -pole given in Ref.[9],

$$\begin{aligned}\alpha^{-1}(m_Z)|_{\text{SM}} &= 127.9 \pm 0.1, \\ \sin^2 \theta_W(m_Z)|_{\text{SM}} &= 0.2317 \pm 0.0003 \pm 0.0002, \\ \alpha_3(m_Z)|_{\text{SM}} &= 0.116 \pm 0.005,\end{aligned}\tag{14}$$

where the index SM means that these values are defined in the framework of the SM. Here, the second error in the Weinberg angle arises from the ambiguity of the Higgs mass (m_h) of the SM, and it has been taken from 60 GeV (−) to 1 TeV (+). In the minimum supersymmetric standard model (MSSM) the mass of the SM-like Higgs is expected to be small (≤ 150 GeV [12]). Thus, we use the Weinberg angle corresponding to the case of $m_h = 60$ GeV in the present analysis.⁴ Since we adopt the $\overline{\text{DR}}$ -scheme (a dimensional

³In the case of $m_{\tilde{h}} \gg m_W, m_{\tilde{w}}$, mixings among gauginos and doublet Higgsinos can be neglected.

⁴When one uses the Weinberg angle with $m_h = 300$ GeV, the upper limit on M_{H_c} is raised up only by factor 1.3.

reduction with the modified minimal subtraction) in our RG analysis, we have to convert the $\overline{\text{MS}}$ coupling constants at the Z -pole into the $\overline{\text{DR}}$ ones;

$$\frac{1}{\alpha_i^{\overline{\text{DR}}}(m_Z)} = \frac{1}{\alpha_i^{\overline{\text{MS}}}(m_Z)} - \frac{C_i}{12\pi}, \quad (15)$$

where $C_1 = 0$, $C_2 = 2$ and $C_3 = 3$ [13].

In our numerical calculations, we derive the constraints on M_{H_c} by using the two-loop RG equations. The decoupling of heavy particles is taken into account at each mass threshold, namely, for $\mu \geq m_{\text{SUSY}}$ we use the RG equations of the MSSM, for $m_{\tilde{g}} \leq \mu \leq m_{\text{SUSY}}$ those of the SM with the wino and the gluino, and so on. Furthermore, we also include the one-loop finite threshold corrections to the gauge coupling constants due to the gaugino loops. Since we assume that the superparticles except for the gauginos are sufficiently heavy, we only have to consider the finite threshold corrections from the gaugino loops to the self energies of the gauge bosons.⁵ In the case $m_{\tilde{g}_i} > m_Z$ ($m_{\tilde{g}_2} = m_{\tilde{w}}$, $m_{\tilde{g}_3} = m_{\tilde{g}}$), we use the following matching condition [14] between the gauge coupling constants α_i in the effective theory defined at $\mu > m_{\tilde{g}_i}$ and those at $\mu < m_{\tilde{g}_i}$,

$$\begin{aligned} \alpha_i^{-1}(m_{\tilde{g}_i} - 0) &= \alpha_i^{-1}(m_{\tilde{g}_i} + 0) \\ &\quad - \frac{C_i}{\pi} \int_0^1 dx \, x(1-x) \ln \left[1 - \frac{m_Z^2}{m_{\tilde{g}_i}^2} x(1-x) \right], \end{aligned} \quad (16)$$

On the contrary, if the wino mass is smaller than the Z -boson mass,⁶ we use the following matching condition [14] for the $\text{SU}(2)_L$ gauge coupling constant;

$$\begin{aligned} \alpha_2^{-1}(m_Z) \Big|_{\text{SM}} &= \alpha_2^{-1}(m_Z) \Big|_{\text{SM}+\tilde{w}} \\ &\quad - \frac{C_2}{\pi} \left\{ \frac{1}{3} \ln \frac{m_{\tilde{w}}}{m_Z} + \int_0^1 dx \, x(1-x) \ln \left[1 - \frac{m_Z^2}{m_{\tilde{w}}^2} x(1-x) \right] \right\}, \end{aligned} \quad (17)$$

while those for $\text{SU}(3)_C$ and $\text{U}(1)_Y$ are $\alpha_i^{-1}(m_Z) \Big|_{\text{SM}} = \alpha_i^{-1}(m_Z) \Big|_{\text{SM}+\tilde{w}}$ ($i = 1, 3$). Here, $\alpha_i^{-1}(m_Z) \Big|_{\text{SM}+\tilde{w}}$ are the gauge coupling constants at $\mu = m_Z$ defined in the SM with the wino. Since we now consider the situation where the gaugino masses are close to m_Z , these finite threshold corrections are not negligible. Notice that the effects of the one-loop finite threshold corrections and the two-loop corrections below the sfermion mass scale have not been included in the previous analysis [7]. Numerically, these corrections decrease the previous limit on M_{H_c} by factor ~ 0.5 .

⁵In the case that the sfermions, the extra Higgs doublet, and the Higgsinos are all heavy, the box and vertex corrections are negligible.

⁶In our analysis, we always use the gluino mass larger than the Z -boson mass since the lower limit $m_{\tilde{g}} \geq 135$ GeV is obtained by the CDF experiments [15].

Next, we comment on the effect of the top-quark Yukawa coupling in the RG equations at the two-loop level. Recently, the CDF collaboration has announced the evidence for top-quark production and reported $m_t = 174 \pm 10_{-12}^{+13}$ GeV [10]. Thus, since the top-quark Yukawa coupling constant y_t is large, its effect should be included [8]. In our numerical analysis, we perform the full integration of the RG equations at the two-loop level and study the effect of the top-quark Yukawa coupling. Here, we take $m_t = 174$ GeV (with m_t being the physical mass of the top quark), and vary $\tan \beta_H$ from 1.3 to 5.⁷ For the case $\tan \beta_H = 1.3$ (2, 5), the effect of top-quark Yukawa coupling raises the upper limit on M_{H_c} by factor 2 (1.3, 1.2).

Combining all the above corrections,⁸ we calculate the upper limit on M_{H_c} . In Fig. 1, the constraints on M_{H_c} are shown in the $m_{\tilde{w}} - M_{H_c}$ plane. As one can see, the upper limit decreases steeply when $m_{\tilde{w}}$ is smaller than $\sim m_Z$ since the finite correction to the $SU(2)_L$ gauge coupling constant given in Eq.(17) is significant in such a region. For the case of a heavy wino ($m_{\tilde{w}} \gtrsim m_Z$), the finite correction given in Eq.(16) from the wino (and the gluino) loop is negligible while the RG effects at the two-loop level lower M_{H_c} slightly. As a result, the upper limit obtained at $m_{\tilde{w}} = 100$ GeV is given by

$$M_{H_c} \leq 2.4 \times 10^{16} \text{ GeV} \quad (90\% \text{ C.L.}), \quad (18)$$

where we have taken $\tan \beta_H = 1.8$.⁹ In Fig. 2, we show the $\tan \beta_H$ dependence of the upper limit on M_{H_c} in the RG analysis, assuming $m_{\tilde{w}} = 100$ GeV. It comes from the effect of the large top-quark Yukawa coupling constant that the upper limit increases in the small $\tan \beta_H$ region. On the other hand, the upper limit on M_{H_c} is almost independent

⁷If $\tan \beta_H$ is smaller than ~ 1.3 , y_t blows up below the GUT scale, while a large value of $\tan \beta_H$ conflicts with the negative search for the proton decay. Therefore, we have taken $\tan \beta_H = 1.3 - 5$. In this case, the effects of the bottom-quark and tau-lepton Yukawa coupling constants are not important, and hence we neglect them below.

⁸Using the 1σ errors of the gauge coupling constants in Eq.(14), the constraint on M_{H_c} is given by

$$M_{H_c} \leq 4.7 \times 10^{15} \text{ GeV},$$

which should be compared with the previous result in Ref.[7]

$$M_{H_c} \leq 2.3 \times 10^{17} \text{ GeV},$$

where the experimental data $\alpha^{-1}(m_Z) = 127.9 \pm 0.2$, $\sin^2 \theta_W(m_Z) = 0.2326 \pm 0.0008$ and $\alpha_3(m_Z) = 0.118 \pm 0.007$ were used. The main reasons that the upper limit on M_{H_c} lowers come from the falloff of the central values of $\sin^2 \theta_W$ and α_3 , and the smaller error bar. Furthermore, the effects of the two-loop corrections below the sfermion mass scale and the one-loop finite corrections give a smaller value in the final result, which have not been taken into account in the previous analysis [7].

⁹As we will see later, the proton-decay rate have its minimum value at $\tan \beta_H \simeq 1.8$.

of $\tan\beta_H$ in the large $\tan\beta_H$ region since the effect of the top-quark Yukawa coupling is small there.

The upper limit on M_{H_c} in Eq.(18) should be compared with the lower limit on M_{H_c} derived from the negative search for the proton decay [16]. The amplitude of the nucleon decay is proportional to the charm- and strange-quark Yukawa coupling constants at the GUT scale,¹⁰ and it takes its minimal value when $\tan\beta_H = 1$ (see Ref.[5]). However, since the top quark is very heavy ($m_t \simeq 174$ GeV), the top-quark Yukawa coupling constant blows up below the GUT scale if $\tan\beta_H \lesssim 1.3$, and hence we can not take $\tan\beta_H = 1$. Moreover, in the RG analysis, the charm-quark Yukawa coupling constant at the GUT scale becomes larger due to the renormalization effects by the large top-quark Yukawa coupling, especially for the case of small $\tan\beta_H$. Therefore, we reanalyze the nucleon decay amplitudes. (In Ref.[5] the top-quark mass has been assumed at 90 GeV.) In Fig. 2 we also show the lower limit on M_{H_c} derived from the negative search for the proton decay in the case of $m_{\tilde{w}} = 100$ GeV. In order to derive a conservative constraint on M_{H_c} , we have taken all sfermion masses at 1TeV, and assumed that the diagrams of scalar-charm and scalar-top exchanges cancel out in the amplitude of the decay mode $p, n \rightarrow K \bar{\nu}_\mu$ and that the dominant decay mode is $p, n \rightarrow \pi \bar{\nu}_\mu$ [5]. The hadron matrix element¹¹ β is obtained from the lattice calculation [18]

$$\beta = (5.6 \pm 0.8) \times 10^{-3} \text{ GeV}^3. \quad (19)$$

To give a conservative constraint, we have used $\beta = 0.004 \text{ GeV}^3$ taking the 2σ errors in Eq.(19). Therefore, the obtained lower limit on M_{H_c} should be regarded as a very conservative one. From Fig. 2, we read off the lower limit on M_{H_c} as

$$M_{H_c} \geq 2.0 \times 10^{16} \text{ GeV}, \quad (20)$$

which corresponds to $\tan\beta_H \simeq 1.8$. In Fig. 1, we also show the lower limit on M_{H_c} in the $m_{\tilde{w}} - M_{H_c}$ plane. From Figs. 1 and 2, we can see that the minimum SUSY-SU(5) model

¹⁰In order to evaluate the strange-quark Yukawa coupling constant at the GUT scale, we have used the strange-quark mass at 1GeV, $m_s = 175\text{MeV}$ [17]. However, if one uses the muon mass $m_\mu = 106\text{MeV}$ instead, one gets almost three times larger amplitude for the proton decay. From this point also the present limit on M_{H_c} is regarded as a conservative one.

¹¹The hadron matrix element β is defined as

$$\beta u_L(\mathbf{k}) \equiv \epsilon_{abc} \langle 0 | (d_L^a u_L^b) u_L^c | p_{\mathbf{k}} \rangle,$$

where $u_L(\mathbf{k})$ is the wave function of the proton with a momentum \mathbf{k} , u_L and d_L are the field operators for the up and down quarks, and $|p_{\mathbf{k}}\rangle$ represents the one-particle state of the proton with a momentum \mathbf{k} . Here, a , b and c are the color indices which run 1 – 3.

is severely constrained, and there remains only a very narrow parameter region.¹²

So far we have assumed that $M_\Sigma \sim M_V \sim 10^{16}$ GeV. However, the two-loop correction from the physical Σ -Higgs loop may loosen the limit on M_{H_c} given in Eq.(18) if the M_Σ is much smaller than M_{H_c} ($\simeq 10^{16}$ GeV). From the RG analysis, M_Σ and M_V are constrained only in the combination as $(M_V^2 M_\Sigma)^{1/3} \simeq 10^{16}$ GeV, as it has been shown in Ref.[7]. Thus, M_Σ and M_V may split largely under the above constraint. For the case $M_\Sigma \gg M_V$ ($M_\Sigma \sim fV$ and $M_V \sim g_5 V$), the Yukawa coupling constant f is so large that it blows up below the gravitational scale. We consider, therefore, the case $M_\Sigma \ll M_V$.¹³ In this case, in the range, $M_\Sigma \leq \mu \leq \min(M_{H_c}, M_V)$, the theory is effectively described by the MSSM with the Σ -Higgs of a mass M_Σ . (Here, we use the notation Σ as the components **(8, 1)** and **(1, 3)** in the $\Sigma(\mathbf{24})$.) Then, the Σ -loop may raise the upper limit on M_{H_c} at the two loop level. In the MSSM with the Σ -Higgs, the two-loop RG equations for the gauge coupling constants are given by¹⁴

$$\mu \frac{d\alpha_i^{-1}}{d\mu} = -\frac{1}{2\pi} \left[b_i + \frac{1}{4\pi} \left(\sum_{j=1}^3 b_{ij} \alpha_j - a_{it} \alpha_t \right) \right] \quad (i = 1, 2, 3), \quad (21)$$

with

$$b_i = b_i^{\text{MSSM}} + \begin{pmatrix} 0, & 2, & 3 \end{pmatrix}^{\Sigma\text{-Higgs}}, \quad (22)$$

$$b_{ij} = b_{ij}^{\text{MSSM}} + \begin{pmatrix} 0 & & \\ & 24 & \\ & & 54 \end{pmatrix}^{\Sigma\text{-Higgs}}, \quad (23)$$

$$a_{it} = a_{it}^{\text{MSSM}}, \quad (24)$$

where $\alpha_t = y_t^2/4\pi$, $(\dots)^{\text{MSSM}}$ is the contribution of MSSM and $(\dots)^{\Sigma\text{-Higgs}}$ that of Σ -Higgs. For the extreme case, $M_\Sigma = 10^{13}$ GeV and $M_V \simeq 10^{18}$ GeV, we obtain the limit on M_{H_c} as

$$M_{H_c} \leq 3.7 \times 10^{16} \text{ GeV} \quad (90 \% \text{ C.L.}). \quad (25)$$

¹²Note that the proton-decay amplitudes are very sensitive to the details of the GUT model. Though the minimum SUSY-SU(5) model is very strongly constrained by the experiments, there is a model in which the proton-decay rate is suppressed naturally [19].

¹³In this extreme case we can solve the serious Polonyi problem [20] in supergravity as stressed in Ref. [21].

¹⁴Strictly speaking, there are more contributions from Yukawa couplings $fTr\Sigma^3$ and $\lambda\bar{H}\Sigma H$ in the two-loop RG equations. But, in the case $M_\Sigma \ll M_V$ the Yukawa coupling constant f is small. We have checked that the Yukawa coupling constant λ is also negligible as far as λ stays in the perturbative regime below the gravitational scale $\simeq 10^{18}$ GeV.

In deriving Eq.(25), we have taken $m_{\tilde{w}} = 100$ GeV and $\tan \beta_H = 1.8$. These effects raise the upper limit on M_{H_c} only by factor ~ 1.5 compared with the result given in Eq.(18),¹⁵ but the situation does not change much.

In summary, we conclude that the minimum SUSY-SU(5) model is severely constrained and that there survives only a very narrow parameter region. The super-KAMIOKANDE experiment will, therefore, give us a conclusive test on the minimum SUSY-GUT.

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¹⁵We comment on the case when the $\lambda \overline{H} \Sigma H$ coupling constant is large ($\lambda \geq 10$). In this case, we have found that the constraint in Eq.(25) is much weakened and even the larger mass $M_{H_c} \sim 10^{18}$ GeV becomes consistent with the low-energy data on the gauge coupling constants. However, we need to perform non-perturbative analysis on RG equations in order to confirm this scenario.

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Figure Captions

Fig. 1

The upper limit on M_{H_c} from the RG analysis and the lower limit from the negative search for the proton decay as a function of $m_{\tilde{w}}$. The solid line is the upper limit and the dashed line the lower limit. The shaded region is excluded. Here, we take $m_{\tilde{q}} = m_{\tilde{l}} = m_{\tilde{h}} = m_H = 1$ TeV, $\beta = 0.004$ GeV³ and $m_t = 174$ GeV. We choose $\tan\beta_H = 1.8$ in order to minimize the decay rate of proton.

Fig. 2

The upper limit on M_{H_c} from the RG analysis and the lower limit from the negative search for the proton decay as a function of $\tan\beta_H$. The solid line is the upper limit and the dashed line the lower limit. The shaded region is excluded. Here, we take $m_{\tilde{q}} = m_{\tilde{l}} = m_{\tilde{h}} = m_H = 1$ TeV, $\beta = 0.004$ GeV³, $m_t = 174$ GeV and $m_{\tilde{w}} = 100$ GeV.

This figure "fig1-1.png" is available in "png" format from:

<http://arXiv.org/ps/hep-ph/9411298v2>

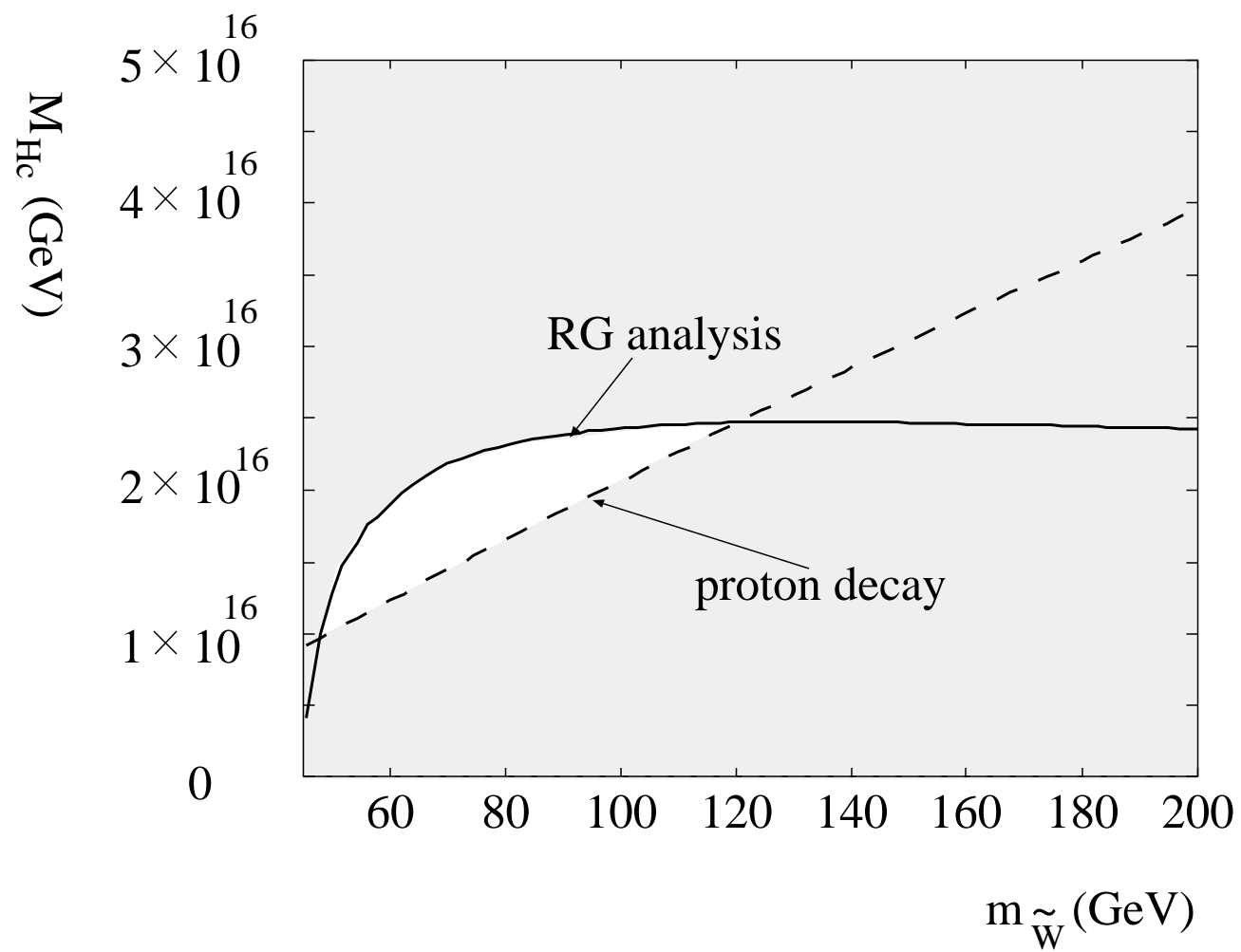


Fig. 1

This figure "fig1-2.png" is available in "png" format from:

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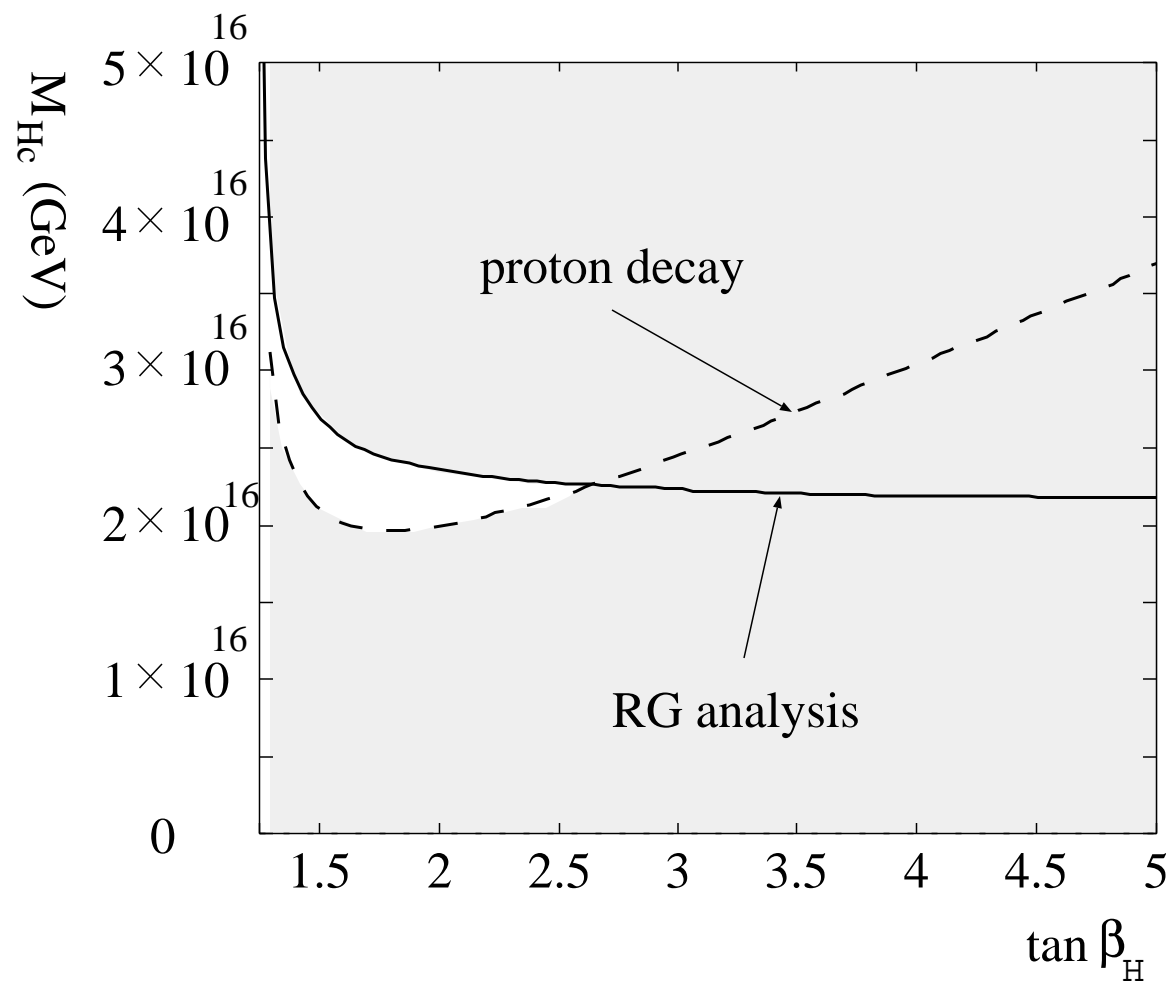


Fig. 2